 revised 03-21-13

(1) Why the expected value of an F-ratio is equal to 1.00 when there is no treatment effect. Numerator of F-ratio estimates between treatment variance (treatment + indiv differences + error). Denominator estimates within treatment variability (indiv. differences + error). When treatment = 0, both numerator & denominator are estimating indiv diff and error variability. Therefore ratio ≈ 1.00

(3) a) As the difference between sample means increases the between treatment variance increases & hence the numerator of F-ratio increases. ∴ F-ratio increases in magnitude.

(6) a) $MS_{Bet} = 0$ (means are the same)

b) F-ratio would be zero

c)

treatments	
1	2
1	2
4	5
0	0
3	1
T = 8	T = 8
SS = 10	SS = 14

$G = 16$
$\sum x^2 = 56$

$df_{bet} = k - 1$
$= 2 - 1$
$= 1$

$$SS_{Bet} = \sum \frac{T^2}{n} - \frac{G^2}{N}$$

$$= \frac{8^2}{4} + \frac{8^2}{4} - \frac{16^2}{8}$$

$$= 16 + 16 - \frac{256}{8}$$

$$= 32 - 32 = 0$$

$$MS_{Bet} = 0/1 = 0$$

11. $MS_{within} = 0$ (no variability within ~~treatment~~ treatment groups)

treatments

I	II
1	3
1	3
1	3
1	3
T=4	T=12

$G = 16$

$\sum X^2 = 40$

12. Relationship between handedness & brain function

Pitch discrimination as a function of handedness (L.H., R.H., ambidextrous)

\# Errors for

R.H.	L.H.	Ambidextrous
6	1	2
4	0	0
3	1	0
4	1	2
3	2	1
T=20	T=5	T=5
SS=6	SS=2	SS=4
$\bar{X}_1 = 4$	$\bar{X}_2 = 1$	$\bar{X}_3 = 1$

$G = 30$

$\sum X^2 = 102$

(12) (cont.)

1.) Set up hypotheses

$H_0: \mu_1 \neq \mu_2 = \mu_3$ (No differences in average # of errors for L.H., R.H., & ambidextrous populations)

H_1: at least one pop. mean is different (There are differences in average # of errors ...)

$\alpha = .05$

2) Set criteria for sampling distribution

$df_{Bet\ Tr.} = k-1 = 3-1 = 2$

$df_{within\ tr.} = N-k = 15-3 = 12$

$df_{TOTAL} = N-1 = 15-1 = 14$

$F_{crit}(2,12) = 3.88$
$\alpha = .05$

3) Compute Sample Statistic →

$F_{obt}(2,12) = \frac{MS_{Bet\ Tr.}}{MS_{within\ Tr.}}$

$= 15.0$

4) Reject H_0 b/c F_{obt} of 15.0 > F_{crit} of 3.88

5) The average number of errors for right-handed, left-handed, & ambidextrous individuals are presented in Table 1. A single factor ANOVA on the number of errors for each group was significant, $F(2,12) = 15.0$, $MSE = 1.00$, $p < .05$.

Source Table

Source	SS	df	MS	F	p<.05
SS_{BT} (handedness)	30	2	15	$F(2,12) = 15.0$	✓
SS_{within}	12	12	1		
	42	14			

$SS_{TOTAL} = \sum x^2 - \frac{G^2}{N} = 102 - \frac{30^2}{15}$

$= \boxed{42}$

$SS_{Bet\ Tr.} = \sum \frac{T^2}{n} - \frac{G^2}{N}$

$= \left(\frac{20^2}{5} + \frac{5^2}{5} + \frac{5^2}{5}\right) - \frac{30^2}{15}$

$= 80 + 5 + 5 - 60 = \boxed{30}$

$SS_{within\ Tr.} = SS_1 + SS_2 + SS_3$

$= 6 + 2 + 4 = \boxed{12}$

(12) (cont.)

$$SD_1 = \sqrt{\frac{SS_1}{df_1}} = \sqrt{\frac{6}{4}} = \sqrt{1.5} = 1.22$$

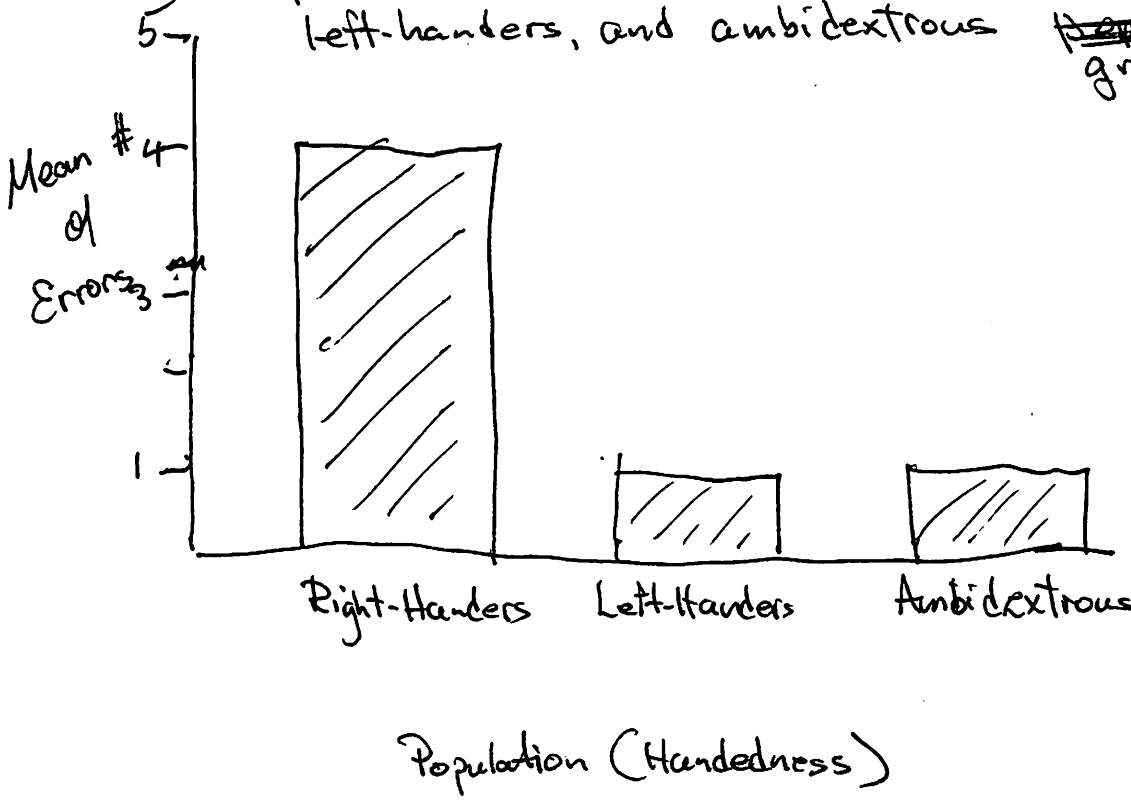
etc.

Table 1. Average number of errors on a pitch discrimination task for different populations

	Population Group		
	Right-Handers	Left-Handers	Ambidextrous
M	4.0	1.0	1.0
SD	1.22	.71	1.41

or (if you did a figure)

Figure 1. Average number of errors on a pitch discrimination task for right-handers, left-handers, and ambidextrous ~~population~~ groups.



(14) 3 samples of rats of $n = 10$

$df_{Bet} = k-1 = 3-1 = 2$

$df_{within} = N-k$
$= 30-3 = 27$

Source	SS	df	MS	
Bet. Treatments	___	2	___	F = 12
Within Treatments	54	27	___	
Total		29		

We know $F = \dfrac{MS_{Bet}}{MS_{within}} = \dfrac{\frac{SS_{Bet}}{df_{Bet.}}}{\frac{SS_{within}}{df_{within}}}$

$\therefore \quad 12 = \dfrac{\frac{SS_{Bet}}{2}}{\frac{54}{27}}$

$12 = \dfrac{SS_{Bet}}{2} \cdot \dfrac{27}{54}$

$12 = \dfrac{SS_B}{2} \cdot \dfrac{1}{2}$

$12 = \dfrac{SS_B}{4}$

$48 = SS_B$

Source	SS	df	MS	
Bet Treat	48	2	24	F = 12
Within Treat	54	27	2	
Total	102	29		

20.

Endomorphs X_1	Ectomorphs X_2	Mesomorphs X_3
23	19	18
25	17	14
19	16	15
20	21	11
23	15	17
$T_1 = 110$	$T_2 = 88$	$T_3 = 75$
$n_1 = 5$	$n_2 = 5$	$n_3 = 5$
$SS_1 = 24$	$SS_2 = 23.2$	$SS_3 = 30$
$\bar{X}_1 = 22$	$\bar{X}_2 = 17.6$	$\bar{X}_3 = 15$

$G = 273$

$N = 15$

$\frac{G^2}{N} = 4968.6$

$\sum x_i^2 = 5171$

$$\boxed{\sum x_i^2 = 23^2 + 25^2 + 19^2 + \ldots 15^2 + 17^2 = 5171}$$

$$SS_1 = \sum x_1^2 - \frac{(\sum x_i)^2}{n_1} = 2444 - \frac{110^2}{5} = 24$$

$$SS_2 = \sum x_2^2 - \frac{(\sum x_2)^2}{n_2} = 19^2 + 17^2 + 16^2 + 21^2 + 15^2 - \frac{88^2}{5} = 23.2$$

$$SS_3 = \sum x_3^2 - \frac{(\sum x_3)^2}{n_3} = 1155 - \frac{75^2}{5} = 30$$

Source	SS	df	MS	F	$p < .05$
Bet. Treat. (Body Type)	125.2	2	62.60	$F(2,12) = 9.74$	✓
Within Treatment	77.2	12	6.43		
Total	202.4 ✓	14 ✓			

Steps

① Set up hypotheses:

$H_0: \mu_1 = \mu_2 = \mu_3$ (There are no differences in sociability scores for different physical types)

H_1: at least one pop. mean is different (There are differences in sociability scores for diff. physical types)

$\alpha = .05$

② Set criteria for testing

$F_{crit}(2, 12) = 3.88$
$\alpha = .05$

$df_{TOTAL} = N - 1 = 15 - 1 = 14$

$df_{Bet\ Tr.} = k - 1 = 3 - 1 = 2$

$df_{within\ Tr.} = N - k = 15 - 3 = 12$

③ Calculate sample statistic

$$F_{obt.}(2,12) = \frac{MS_{Bet\ Tr.}}{MS_{within\ Tr.}} = \text{see source table}$$

$$SS_{TOTAL} = \Sigma x^2 - \frac{G^2}{N} = 5171 - \frac{273^2}{15}$$

$$= 5171 - 4968.6$$

$$= 202.4$$

(3) (cont.)

$$SS_{\text{Bet. Tr.}} = \sum \frac{T^2}{n} - \frac{G^2}{N}$$

$$= \left(\frac{110^2}{5} + \frac{88^2}{5} + \frac{75^2}{5} \right) - \frac{273^2}{15}$$

$$= (2420 + 1548.8 + 1125) - \cancel{\text{[illegible]}} \; 4968.6$$

$$= \boxed{125.2}$$

$$SS_{\text{within tr.}} = \sum SS \text{ within each treatment group}$$

$$= SS_1 + SS_2 + SS_3$$

$$= 24 + 23.2 + 30$$

$$= \boxed{77.2}$$

(4) Decision. Reject H_0 b/c F_{obt} of 9.74 $>$ F_{crit} of 3.88

(5) Conclusion:
Average sociability scores for different body types are presented in Figure 1. A single-factor analysis of variance on the sociability scores was significant, $F(2, 12) = 9.74$, $MSE = 9.74$, $p < .05$.

~~Step 4. Reject H_0 because F_{obt} of 9.74 greater than F_{crit} of 3.88~~

~~Step 5 Conclusion: Average sociability scores were 22, 17.6, and 15 for the endomorphs, ectomorphs, and mesomorphs, respectively. There was an overall effect of body type on sociability scores, $F(2, 12) = 9.74$, $MSE = 6.43$, $p < .05$.~~

23. Quality ratings as a function of physical attractiveness:

Attractive			Average			Unattractive		
5	4	4	6	5	3	4	3	1
3	5	6	6	6	7	3	1	2
4	3	8	5	4	6	2	4	3
3	5	4	8	7	8	2	1	2

$G = 153$

$\sum x^2 = 789$

$N = 36$

Attractive	Average	Unattractive
$T_1 = 54$	$T_2 = 71$	$T_3 = 28$
$n_1 = 12$	$n_2 = 12$	$n_3 = 12$
$\bar{X}_1 = 4.5$	$\bar{X}_2 = 5.9$	$\bar{X}_3 = 2.3$
$SS_1 = 23$	$SS_2 = 25$	$SS_3 = 13$

$$SS_1 = \sum x^2 - \frac{(\sum x)^2}{n} = 266 - \frac{54^2}{12} = 266 - 243 = \boxed{23}$$

$$SS_2 = \sum x^2 - \frac{(\sum x)^2}{n} = 445 - \frac{71^2}{12} = 445 - 420 = \boxed{25}$$

$$SS_3 = \sum x^2 - \frac{(\sum x)^2}{n} = 78 - \frac{28^2}{12} = 78 - 65 = 13$$

Source	SS	df	MS	F	$p<.05$
Bet Treatments	77.75	2	38.88	$F(2,33)=21.02$	✓
Within Treatments	61	33	1.85		
Total	138.75	35			

Step 1: State hypotheses

$H_0: \mu_1 = \mu_2 = \mu_3$ (No effect of physical attractiveness on applicant quality ratings)

H_1: At least one pop mean is diff. (There is an effect of physical...)

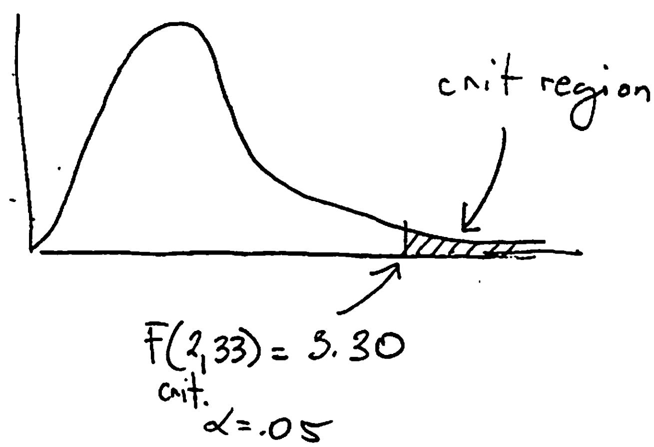
$\alpha = .05$

Step 2: Define the crit region

$df_{TOT} = N-1 = 36-1 = 35$

$df_{Bet.} = K-1 = 3-1 = 2$

$df_{within} = N-K = 36-3 = 33$



Step 3: $F_{obt.}(2,33) = \dfrac{MS_{Bet.}}{MS_{within}}$

$$SS_{TOTAL} = \sum x^2 - \frac{G^2}{N} = 789 - \frac{153^2}{36}$$

$$= 789 - \frac{23409}{36}$$

$$= 789 - 650.25$$

$$= \boxed{138.75}$$

$$SS_{Bet.} = \sum \frac{T^2}{n} - \frac{G^2}{N} = \left(\frac{54^2}{12} + \frac{71^2}{12} + \frac{28^2}{12}\right) - \frac{153^2}{36}$$

$$= (243 + 420 + 65) - 650.25$$

$$= 728 - 650.25$$

$$= \boxed{77.75}$$

$$SS_{within} = SS_1 + SS_2 + SS_3$$

$$= 23 + 25 + 13$$

$$= \boxed{61}$$

$$F_{obt}(2, 33) = \frac{MS_B}{MS_W} = \boxed{21.02}$$

$$SS_T = SS_B + SS_W$$

$$138.75 = 77.75 + 61$$

$$= 138.75 \quad \checkmark$$

Step 4: Reject H_0 because F_{obt} of 21.02 exceeds F_{crit} of 3.30

Step 5: Conclusion. Average job applicant ratings as a function of physical attractiveness are presented in Figure 1. There was a significant effect of attractiveness on quality ratings, $F(2, 33) = 21.02$, $MSE = 1.85$, $p < .05$