1. RM design uses fewer subjects (participants) and is more likely to detect a treatment effect (if there is one) — this is because sampling error due to indiv. diffs has been eliminated

2. For independent-measures ANOVA, the variability within treatments (MS_{within}) is the appropriate error term. For repeated-measures ANOVA, the appropriate error term is the variability due to experimental error (or MS_{error}), which is obtained by subtracting the variability due to individual differences from the variability within

4.) F ratio with df = 3, 36   RM ANOVA

a) Treatment conditions was $K = 4$

b) Number of participants must have been $n = 13$ since lose 1 df for each treatment

Because $(K-1)(n-1) = N-K-(n-1)$

or

Source	SS	df	MS
Bet treat		3	
Within treat		N-K	
(Bet Subj.)		(n-1)	
(Error)		36	
Total		N-1	

$$\therefore \quad (K-1)(n-1) = 36$$

$$3(n-1) = 36 \rightarrow \boxed{\therefore n = 13}$$

$$3n - 3 = 36$$

$$3n = 39$$

$$\boxed{n = 13}$$

6)

Session

RAT	1	2	3	4	P
1	3	1	0	0	4
2	3	2	2	1	8
3	6	3	1	2	12
	$T_1 = 12$	$T_2 = 6$	$T_3 = 3$	$T_4 = 3$	$G = 24$
	$n_1 = 3$	$n_2 = 3$	$n_3 = 3$	$n_4 = 3$	$G^2 = 576$
	$SS_1 = 6$	$SS_2 = 2$	$SS_3 = 2$	$SS_4 = 2$	$\Sigma X^2 = 78$
	$\bar{X}_1 = 4$	$\bar{X}_2 = 2$	$\bar{X}_3 = 1$	$\bar{X}_4 = 1$	

$$SS_1 = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = (3^2 + 3^2 + 6^2) - \frac{(12)^2}{3}$$

$$= (9+9+36) - \frac{144}{3} = 54 - 48 = \boxed{6}$$

$$SS_2 = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = (1^2 + 2^2 + 3^2) - \frac{6^2}{3} = (1+4+9) - \frac{36}{3}$$

$$= 14 - 12 = \boxed{2}$$

$$SS_3 = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = (0^2 + 2^2 + 1^2) - \frac{3^2}{3} = 5 - 3 = \boxed{2}$$

$$SS_4 = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = \boxed{2}$$

Start Source table

Source	SS	df	MS	F
Bet treat	18	3	6.0	$F(3,6) = 8.96$
Within treat	12	8		
Bet Sus.	(8)	(2)		
Error	(4)	(6)	.67	
Total	30	11		

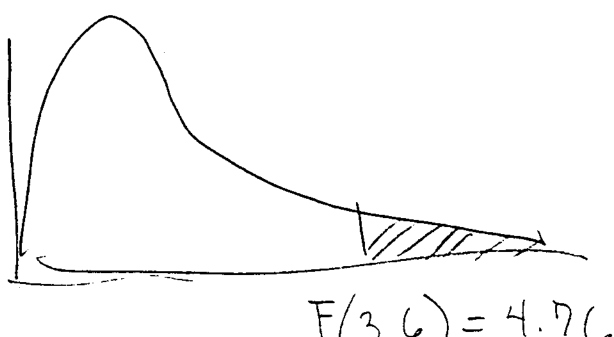
)

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$ (There is no effect of practice on # errors)

H_1: at least one mean is different. (There <u>is</u> an effect of practice on # errors)

$\alpha = .05$

) Set. Criteria



$F_{crit}(3,6) = 4.76$

$F_{crit}(3,6) = 4.76$

$df_{TOTAL} \quad N-1 = 12-1 = 11$

$df_{Bet.\,treat.} = k-1 = 4-1 = 3$

$df_{with.\,treat.} = N-k = 12-4 = 8$. add ✓

$df_{Bet.\,Subj.} = n-1 = 3-1 = 2$

$\therefore\ df_{error} = df_{within} - df_{Bet\,Subj.} = 8-2 = \boxed{6}$

or $\quad df_{error} = (df_{Bet.\,treat})(df_{Bet.\,tr.}) = (3)(2) = \boxed{6}$ ✓

ep. 3) Sample statistic

$$F_{obt.}(3,6) = \frac{MS_{Bet\,tr.}}{MS_{error}} = \frac{\dfrac{SS_{Bet\,tr.}}{df_{Bet\,tr.}}}{\dfrac{SS_{error}}{df_{error}}} = \frac{\frac{18}{3}}{\frac{4}{6}} = \frac{6}{.667} = \boxed{8.96}$$

$$SS_{TOTAL} = \sum X^2 - \frac{G^2}{N} = 78 - \frac{576}{12} = 78 - 48 = \boxed{30}$$

$$SS_{Bet.} = \sum \frac{T^2}{n} - \frac{G^2}{N} = \left(\frac{12^2}{3} + \frac{6^2}{3} + \frac{3^2}{3} + \frac{3^2}{3}\right) - 48$$

$$= (48 + 12 + 3 + 3) - 48 = \boxed{18}$$

$SS_{within} = \sum SS_{\text{within each treatment}}$

$= 6 + 2 + 2 + 2 = \boxed{12}$

$SS_{Bet\ Sub.} = \sum \frac{P^2}{K} - \frac{G^2}{N} = \left(\frac{4^2}{4} + \frac{8^2}{4} + \frac{12^2}{4}\right) - 48$

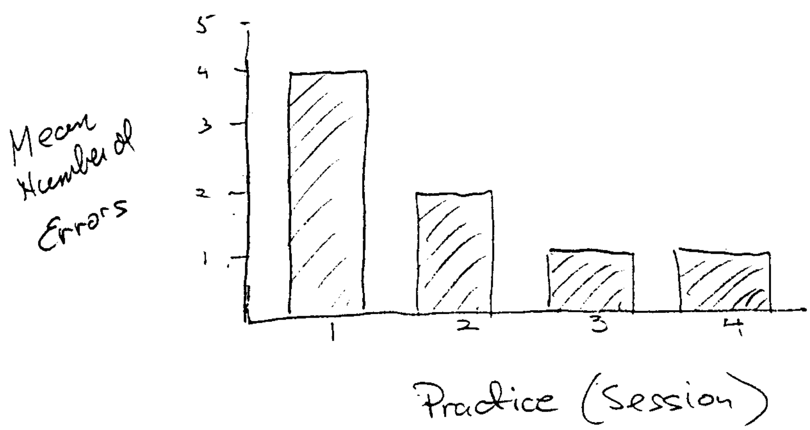
$= (4 + 16 + 36) - 48 = 56 - 48$

$= \boxed{8}$

$SS_{error} = SS_{within} - SS_{Bet.S} = 12 - 8 = \boxed{4}$

Step 4 Reject H_0 because $F_{obt.}(3,6)$ of 8.96 > F_{crit} of 4.76

Step 5: Conclusion. The average number of errors for the four sessions are presented in figure 1. The overall effect of practice on errors was significant, $F(3,6) = 8.96$, $MSE = .67$, $p < .05$



HSD post hoc tests

$p < .05$

$$HSD = q\sqrt{\frac{MS_{error}}{n}} =$$

$k = 4$, $df_{within} = 6$

$q = 4.90$

$$= (4.90)\sqrt{\frac{.67}{3}} =$$

$$= (4.90)\sqrt{.22333}$$

$$= (4.90)(.4726) = \boxed{2.32}$$

∴ There was a significant difference in mean number of errors for session 1 ($M = 4$) compared to sessions 3 ($M = 1$) and session 4 ($M = 1$). None of the other comparisons were significant.

10. Serial Position

Person	First	Middle	Last	P
A	1	5	0	6
B	3	7	2	12
C	5	6	1	12
D	3	2	1	6
	$T_1 = 12$	$T_2 = 20$	$T_3 = 4$	$G = 36$
	$n_1 = 4$	$n_2 = 4$	$n_3 = 4$	$G^2 = 1296$
	$\bar{X}_1 = 3$	$\bar{X}_2 = 5$	$\bar{X}_3 = 1$	$\sum X^2 = 164$
	$SS_1 = 8$	$SS_2 = 14$	$SS_3 = 2$	$N = 12$

$$SS_1 = \sum X^2 - \frac{(\sum X)^2}{n} = (1^2 + 3^2 + 5^2 + 3^2) - \frac{12^2}{4}$$

$$= (1 + 9 + 25 + 9) - \frac{144}{4}$$

$$= 44 - 36 = \boxed{8}$$

$$SS_2 = \sum X^2 - \frac{(\sum X)^2}{n} = (5^2 + 7^2 + 6^2 + 2^2) - \frac{20^2}{4}$$

$$= (25 + 49 + 36 + 4) - \frac{400}{4}$$

$$= 114 - 100 = \boxed{14}$$

$$SS_3 = \sum X^2 - \frac{(\sum X)^2}{n} = (0^2 + 2^2 + 1^2 + 1^2) - \frac{4^2}{4}$$

$$= (0 + 4 + 1 + 1) - 4$$

$$= 6 - 4 = \boxed{2}$$

Start Source Table:

Source	SS	df	MS	F
Bet. Treat.	32	2	160	$F(2,6) = 8.00$
Within Tr.	24	9		
Bet. Subj.	(12)	(3)		
Error	(12)	(6)	2.0	
Total	56	11		

$\boxed{\text{Step 1}}$

$H_0: \mu_1 = \mu_2 = \mu_3$ (There are no differences in average number of errors for different serial positions)

H_1: at least one pop mean is diff. (There is some difference in average # of errors for the different serial positions)

$\alpha = .05$

$\boxed{\text{Step 2}}$ Set criteria - need df

$F_{crit}(2,6) = 5.14$

$df_{TOTAL} = N - 1 = 12 - 1 = \boxed{11}$

$df_{Bet. Tr.} = k - 1 = 3 - 1 = \boxed{2}$

$df_{within\ tr.} = N - k = 12 - 3 = \boxed{9}$ ✓✓

$df_{Bet. subj.} = n_s - 1 = 4 - 1 = \boxed{3}$

$df_{error} = df_{within} - df_{subj.} = 9 - 3 = \boxed{6}$

or $df_{error} = (df_{Bet. tr.})(df_{Bet. Subj.}) = (2)(3) = \boxed{6}$

Step 3 Sample statistic

$$F(2,6) = \frac{MS_{Bet.}}{MS_{error}} = \frac{\frac{SS_{Bet. Tr}}{df_{Bet. Tr.}}}{\frac{SS_{error}}{df_{error}}}$$

$$= \frac{\frac{32}{2}}{\frac{6}{2}} = \frac{16.0}{2.0} = \boxed{8.00}$$

$$SS_{TOTAL} = \sum X^2 - \frac{G^2}{N} = 164 - \frac{1296}{12}$$

$$= 164 - 108 = \boxed{56}$$

$$SS_{Bet\ tr.} = \sum \frac{T^2}{n} - \frac{G^2}{N} = \left(\frac{12^2}{4} + \frac{20^2}{4} + \frac{4^2}{4} \right) - 108$$

$$= (36 + 100 + 4) - 108 = \boxed{32}$$

$SS_{\text{within tr.}} = \sum SS_{\text{within each treatment}}$

$= 8 + 14 + 2 = \boxed{24}$

$SS_{\text{Bet. Subj.}} = \sum \frac{P^2}{K} - \frac{G^2}{N} = \left(\frac{6^2}{3} - \frac{12^2}{3} + \frac{12^2}{3} + \frac{6^2}{3}\right) - 128$

$= (12 + 48 + 48 + 12) - 108$

$= \boxed{12}$

$SS_{\text{Error}} = SS_{\text{within}} - SS_{\text{Bet. Subj.}} = 24 - 12 = \boxed{12}$

Step 4 Reject H_0 because $F_{\text{obt.}}$ of 8.00 > F_{crit} of 5.14

Step 5 The average number of errors for the first, middle, and last serial positions were 3, 5, and 1 errors respectively. There was an overall significant effect of serial position on average number of errors, $F(2,6) = 8.00$, $MSE = 2.0$, $p < .05$

(1c) cont. Post Hoc

p < .05

$$HSD = q\sqrt{\frac{MS_{error}}{n}}$$

$k = 3$

$df_{error} = 6$

$q = 4.34$

$$= 4.34\sqrt{\frac{2.0}{4}}$$

$$= 4.34\sqrt{.5} = (4.34)(.707)$$

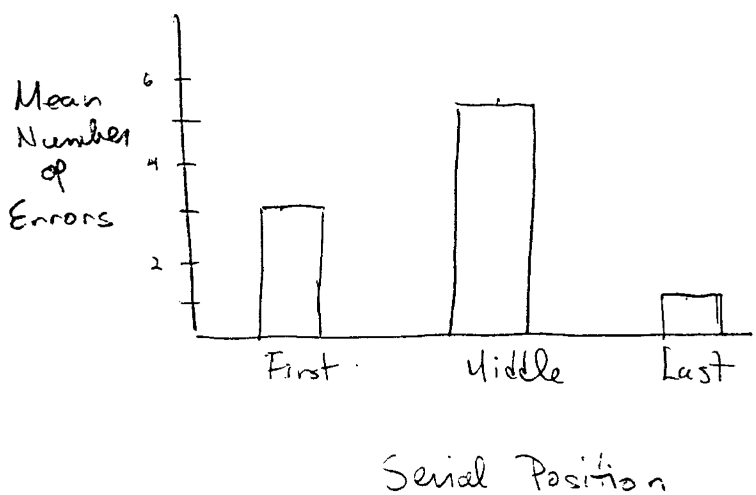
$$= \boxed{3.1}$$

∴

1st vs. Middle	5 − 3 = 2	n.s.	
1st vs. Last	3 − 1 = 2	n.s.	
Middle vs. Last	5 − 1 = 4	sig.	p<.05

There was a significant difference in the mean number of errors for the middle serial position ($M = 5$) versus the last serial position ($M = 1$). None of the other differences were significant.

Graph



(16.) Effectiveness of Reading Skills Course on Comprehension

sample of $n = 15$ students $n-1 = 15-1 = \boxed{14}$

Source	SS	df	MS	
Betw. Trts.	?	?	24	F = 8
Within Trtmnts	120	?		
Bet Ss		?		
Error	?	?	?	
Total	?	?		

$k = 3$

$\therefore df_{Bet.} = 3-1 = 2$

↓

Source	SS	df	MS	
Bet. T	48	2	24	F = 8
Within. T	120	42		
Bet. Subj.	36	14		
Error	84	28	3	
Total	168	44		

16) cont.

$$F = \frac{MS_{Bet.}}{MS_{error}}$$

$$\therefore \quad 8 = \frac{24}{MS_{error}} \Longrightarrow MS_{error} = \boxed{3}$$

$$df_{Bet.\,Sub.} = n-1 = 15-1 = 14$$

$$SS_{TOT} = SS_{Bet.\,tr.} + SS_{within} = 48 + 120 = \boxed{168}$$

Total # of observations $= 3 \times 15 = 45$

$$\therefore \quad df_{TOT} = N-1 = 45-1 = \boxed{44}$$

$df_{within} = ?$

$$df_{TOT} = df_{Bet.\,treat} + df_{within}$$

$$44 = 2 + df_{within}$$

$$\therefore \quad df_{within} = \boxed{42}$$

$$df_{error} = df_{within} - df_{Bet\,Sub.} = 42 - 14 = \boxed{28}$$

– $MS_{error} = \dfrac{SS_{error}}{df_{error}}$

$$3 = \frac{SS_{error}}{28} \quad \therefore \; SS_{error} = \boxed{24}$$

– $SS_{within} = SS_{Bet.\,Subj.} + SS_{error}$

$$120 = SS_{Bet.\,Subj.} + 84$$

$$\therefore \; SS_{Bet.\,Subj.} = 36$$

(19.) Educational Psychologist studying motivation
sample of $n = 5$ students from 4^{th} - 6^{th} grade

Motivation Level Scores

Student	Fourth Grade	Fifth Grade	Sixth Grade	P
A	4	3	1	8
B	8	6	4	18
C	5	3	3	11
D	7	4	2	13
E	6	4	0	10
	$T_1 = 30$	$T_2 = 20$	$T_3 = 10$	$G = 60$
	$n_1 = 5$	$n_2 = 5$	$n_3 = 5$	$G^2 = 3600$
	$\bar{X}_1 = 6$	$\bar{X}_2 = 4$	$\bar{X}_3 = 2$	$\sum X^2 = 306$
	$SS_1 = 10$	$SS_2 = 6$	$SS_3 = 10$	$N = 15$

$$SS_1 = \sum x^2 - \frac{(\sum x)^2}{n} = (4^2 + 8^2 + 5^2 + 7^2 + 6^2) - \frac{30^2}{5}$$
$$= (16 + 64 + 25 + 49 + 36) - 180 = 190 - 180 = \boxed{10}$$

$$SS_2 = \sum x^2 - \frac{(\sum x)^2}{n} = (3^2 + 6^2 + 3^2 + 4^2 + 4^2) - \frac{20^2}{5}$$
$$= (9 + 36 + 9 + 16 + 16) - 80 = 86 - 80 = \boxed{6}$$

$$SS_3 = \sum x^2 - \frac{(\sum x)^2}{n} = (1^2 + 4^2 + 3^2 + 2^2 + 0^2) - \frac{10^2}{5}$$
$$= (1 + 16 + 9 + 4 + 0) - 20$$
$$= 30 - 20 = \boxed{10}$$

Stat Source Table

Source	SS	df	MS	F
Bet. Treat	40	2	20.0	$F(2,8) = 23.95$
Within Treat.	26	12		
(Bet. Subj.)	(19.32)	(4)		
(Error)	(6.68)	(8)	.835	
Total	66	14		

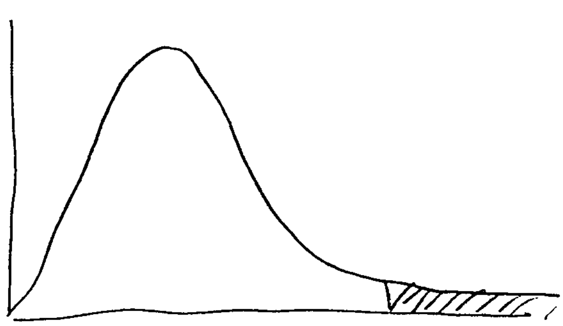
Step 1 State Hypotheses

$H_0: \mu_1 = \mu_2 = \mu_3$ (There are no differences in motivation level across grade level)

H_1: At least one pop. mean is diff. (There are changes in motivation level across grade level)

$\alpha = .05$

Step 2 Set criteria



$F_{crit}(\quad) =$

$F_{crit}(2,8) = \boxed{4.46}$

Compute df to find

$$df_{TOTAL} = N-1 = 15-1 = \boxed{14}$$

$$df_{Bet.Tr.} = k-1 = 3-1 = \boxed{2}$$

$$df_{\text{within Treatments}} = N-K = 15-3 = \boxed{12}$$ ✓✓

$$df_{Bet.\ Subj} = n_s - 1 = 5-1 = \boxed{4}$$

$$df_{error} = df_{within} - df_{Bet.\ Subj.} = 12-4 = \boxed{8}$$

or

$$df_{error} = (df_{Bet.\ Treat.})(df_{Bet.\ Subj}) = (2)(4) = \boxed{8}$$

Step 3 Compute sample statistic

$$F = \frac{MS_{Bet.\ Treat.}}{MS_{error}} = \frac{\frac{SS_{Bet.tr.}}{df_{Bet.Tr.}}}{\frac{SS_{error}}{df_{error}}} = \frac{\frac{40}{2}}{\frac{6.68}{8}} = \frac{20}{.835} = \boxed{23.95}$$

$$SS_{TOTAL} = \Sigma x^2 - \frac{G^2}{N} = 306 - \frac{3600}{15} = 306 - 240 = \boxed{66}$$

$$SS_{Bet.\ Treat.} = \Sigma \frac{T^2}{n} - \frac{G^2}{N} = \left(\frac{30^2}{5} + \frac{20^2}{5} + \frac{10^2}{5}\right) - 240$$

$$= (180 + 80 + 20) - 240$$

$$= 280 - 240 = \boxed{40}$$

Step 3' cont.

$SS_{within} = \sum SS_{\text{within each treatment}}$ check sums ✓✓

$= 10 + 6 + 10 = \boxed{26}$

$$SS_{Bet.Subj.} = \sum \frac{P^2}{K} - \frac{G^2}{N} =$$

$$= \left(\frac{8^2}{3} + \frac{18^2}{3} + \frac{11^2}{3} + \frac{13^2}{3} + \frac{10^2}{3}\right) - 240$$

$$= (21.33 + 108 + 40.33 + 56.33 + 33.33) - 240$$

$$= 259.32 - 240 = \boxed{19.32}$$

$$SS_{error} = SS_{within} - SS_{Bet.Subj.} = 26 - 19.32 = \boxed{6.68}$$

Step4 Decision Reject H_0 because

F_{obt} of 23.95 > F_{crit} of 4.46

Step 5 Conclusion. There were differences in the motivation levels for the fourth graders ($M = 6$), the fifth graders ($M = 4$) and the sixth graders ($M = 2$). The overall effect of grade level on motivation level was significant, $F(2, 8) = 23.95$, $MSE = .835$, $p < .05$.

Post hoc tests using Tukey's HSD

$$HSD = q\sqrt{\frac{MS_{error}}{n}}$$

$K = 3$

$df_{error} = 8$

$\therefore q = 4.04$

$$= 4.04\sqrt{\frac{.835}{5}}$$

$$= 4.04\sqrt{.167}$$

$$= (4.04)(.4087) = \boxed{1.65}$$

minimum diff necc. for signif.

$\therefore$ 1) 4th grade ($M=6$) vs. 5th ($M=4$) signif, $p<.05$
2) 4th grade ($M=6$) vs. 6th ($M=2$) signif, $p<.05$
3) 5th grade ($M=4$) vs. 6th ($M=2$) signif, $p<.05$

Using an HSD of 1.65, $p<.05$, to compare the means, we found the average motivation level for the fourth grade class ($M=6$) was significantly different from that for the 5th grade class ($M=4$) and from that the 6th grader class ($M=2$). In addition, the average motivation level for the 5th graders ($M=4$) was significantly different from the average motivation level for the 6th grade class ($M=2$).