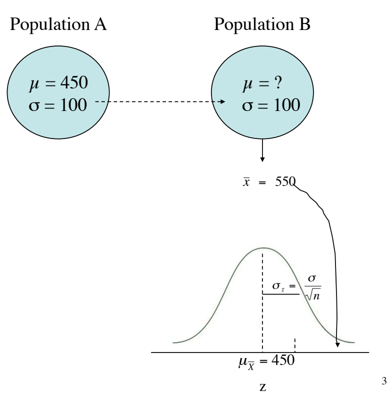
Chapter 10: The t Test For Two Independent Samples

Independent-Measures Designs

- The independent-measures hypothesis test allows researchers to evaluate the mean difference between two populations using the data from two separate samples.
- The identifying characteristic of the **independent-measures** or **between-subjects** design is the existence of two separate or independent samples.
- Thus, an independent-measures design can be used to test for mean differences between two distinct populations (such as men versus women) or between two different treatment conditions (such as drug versus no-drug).



3

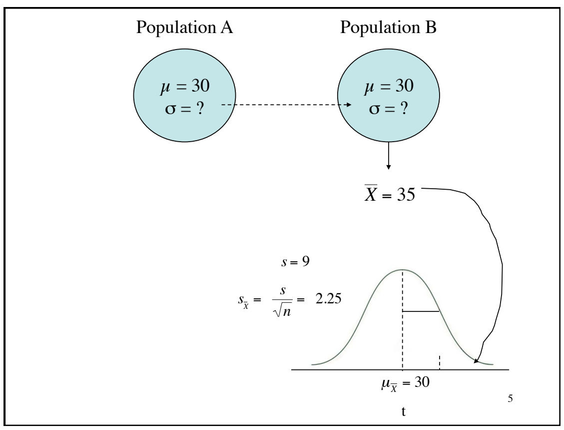
Sample Statistic

Population Parameter

$$z = \frac{\overline{X} - \mu_{\overline{X}}}{\sigma_{\overline{X}}}$$

Standard Error
(Sampling Error)

4



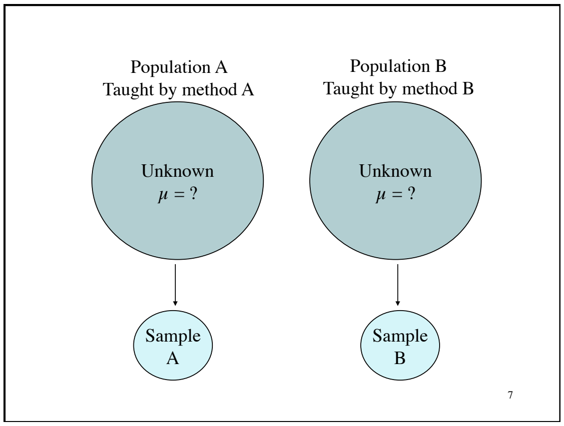
Sample Statistic

Population Parameter

$$t = \frac{\overline{X} - \mu_{\overline{X}}}{s_{\overline{X}}}$$

Estimated Standard Error
(Sampling Error)

6

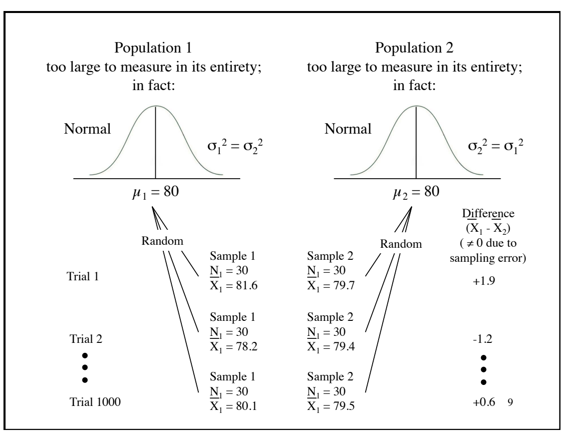


Goal: Evaluate Mean difference Between 2 Population Means

H_o: $\mu_1 - \mu_2 = 0$ (No difference between the population means)

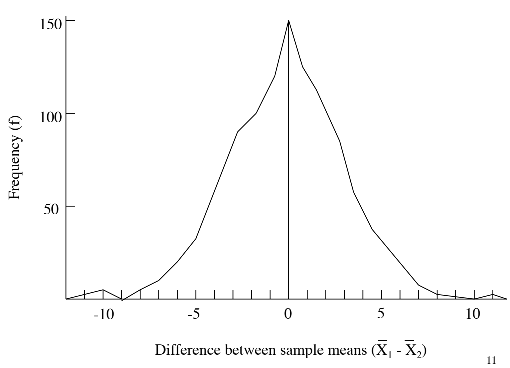
H_1: $\mu_1 - \mu_2 \neq 0$ (There is a difference between the pop means)

8



Difference Between Samples Means ($\bar{X}_1 - \bar{X}_2$)	Number of Samples
Greater than +11.49	0
+10.50 to +11.49	1
+9.50 to +10.49	0
+8.50 to +9.49	1
+7.50 to +8.49	4
+6.50 to +7.49	7
+5.50 to +6.49	21
+4.50 to +5.49	32
+3.50 to +4.49	54
+2.50 to +3.49	77
+1.50 to +2.49	107
+.50 to +1.49	122
-.50 to +.49	153
-1.50 to -.51	114
-2.50 to -1.51	95
-3.50 to -2.51	82
-4.50 to -3.51	60
-5.50 to -4.51	31
-6.50 to -5.51	22
-7.50 to -6.51	10
-8.50 to -7.51	4
-9.50 to -8.51	1
-10.50 to -9.51	2
Smaller than --10.51	0
	Total 1000

10



11

Sample Statistic

Population Parameter

$$t = \frac{\bar{X} - \mu_{\bar{X}}}{s_{\bar{X}}}$$

Estimated Standard Error (Sampling Error)

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{s_{\bar{X}_1 - \bar{X}_2}}$$

12

Sample Statistic

Population Parameter

$$t = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{s_{\overline{X}_1 - \overline{X}_2}}$$

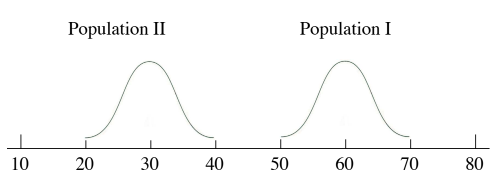
Sampling Error
(Standard Error of the Mean Difference)

13

$$S_{\overline{X}} = \sqrt{\frac{s^2}{n}} \quad or \quad \frac{s}{\sqrt{n}}$$

$$S_{\overline{X}_1 - \overline{X}_2} = \sqrt{\frac{s_1^{\,2}}{n_1} + \frac{s_2^{\,2}}{n_2}}$$

14



15

$$S_{\bar{X}_1-\bar{X}_2} = \sqrt{\frac{s_p^{\ 2}}{n_1} + \frac{s_p^{\ 2}}{n_2}}$$

$$\boxed{S_p^{\ 2} = \frac{df_1 s_1^{\ 2} + df_2 s_2^{\ 2}}{df_1 + df_2}}$$

OR

$$\boxed{S_p^{\ 2} = \frac{SS_1 + SS_2}{df_1 + df_2}}$$

16

Final Formula for Independent t

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{s_{\bar{X}_1 - \bar{X}_2}}$$

$$= \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_p^{\ 2}}{n_1} + \frac{S_p^{\ 2}}{n_2}}}$$

df = df 1st sample + df 2nd sample

= $df_1 + df_2$

OR df = $(n_1 - 1) + (n_2 - 1)$

17

1. H_o: $\mu_1 - \mu_2 = 0$ (No effect of…)
   H_1: $\mu_1 - \mu_2 \neq 0$ (Effect…)
   α
2. Critical region for t statistic
   $df = df_1 + df_2$ $p < \alpha$
   $t_{critical}$ =
   α =
3. Collect data and compute test statistic
   $$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{S_{\bar{X}_1 - \bar{X}_2}}$$
4. Make a decision
5. Conclusion

18

Do mental images help memory?

- 40 pairs of nouns
  - (dog / bicycle, lamp / piano, etc.)
- 2 groups (or samples)
  - 1. Memorization (control)
  - 2. Imagery (experimental)
- Subsequent <u>memory test</u>

19

Data (Number of words recalled)

Group 1 (No images)		Group 2 (Images)	
24	13	18	31
23	17	19	29
16	20	23	26
17	15	29	21
19	26	30	24

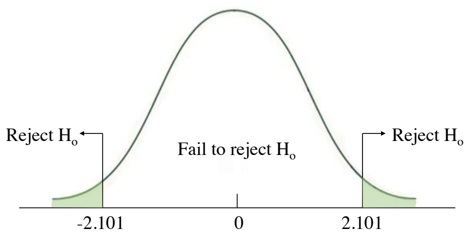
20

Data (Number of words recalled)

Group 1 (No images)		Group 2 (Images)	
24	13	18	31
23	17	19	29
16	20	23	26
17	15	29	21
19	26	30	24

Group 1	Group 2
$n = 10$	$n = 10$
$\overline{X} = 19$	$\overline{X} = 25$
$SS = 160$	$SS = 200$

21



22

$$t = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{s_{\overline{X}_1 - \overline{X}_2}}$$

$$S_{\overline{X}_1 - \overline{X}_2} = \sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}$$

23

$SS_1 = \sum (x - \overline{X}_1)^2$

X_1	$\overline{X}_1$	$(X_1 - \overline{X}_1)$	$(X_1 - \overline{X}_1)^2$
24	19	+5	25
23	19	+4	16
16	19	-3	9
17	19	-2	4
19	19	0	0
13	19	-6	36
17	19	-2	4
20	19	+1	1
15	19	-4	16
26	19	+7	49

$\sum (x - \overline{X})^2 = 160$

24

$$SS_1 = \sum x^2 - \frac{(\sum x)^2}{n}$$

X_1	X_1^2
24	576
23	529
16	256
17	289
19	361
13	169
17	289
20	400
15	225
26	676
$\sum x = 190$	$\sum x^2 = 3770$

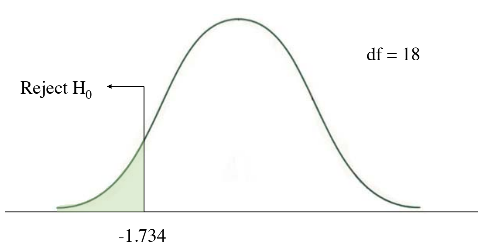
$$SS_1 = \sum x^2 - \frac{(\sum x)^2}{n}$$

$$= 3770 - \frac{190^2}{10}$$

$$= 3770 - 3610$$

$$= 160$$

25



26

Assumptions Underlying the Independent-Measures t statistic

1. The observation within each sample must be independent
2. The two populations from which the samples come must be normal
3. The two populations from which the samples are selected must have equal variances - <u>homogeneity of variance</u>

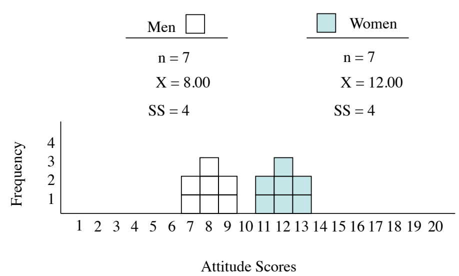
27

Homogeneity Assumption: How do we know when we have violated it?

1. For small samples (n < 10):
   - If one of the sample variances is more than 4 times larger than the other

2. For larger samples:
   - If one of the sample variances is <u>more than</u> 2 times larger than the other

28



29

30