Chapter 4: Variability

Variability

- Provides a quantitative measure of the degree to which scores in a distribution are spread out or clustered together

Central Tendency and Variability

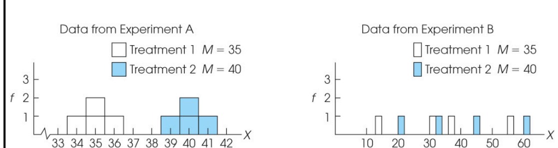
- Central tendency describes the central point of the distribution, and variability describes how the scores are scattered around that central point.
- Together, central tendency and variability are the two primary values that are used to describe a distribution of scores.

Variability

- Variability serves both as a descriptive measure and as an important component of most inferential statistics.
- As **a descriptive statistic**, variability measures the degree to which the scores are spread out or clustered together in a distribution.
- In the context of **inferential statistics**, variability provides a measure of how accurately any individual score or sample represents the entire population.

Variability (cont.)

- When the population variability is small, all of the scores are clustered close together and any individual score or sample will necessarily provide a good representation of the entire set.
- On the other hand, when variability is large and scores are widely spread, it is easy for one or two extreme scores to give a distorted picture of the general population.



Measuring Variability

- Variability can be measured with
  - the range
  - the interquartile range
  - the standard deviation/variance.
- In each case, variability is determined by measuring *distance*.

The Range

- The **range** is the total distance covered by the distribution, from the highest score to the lowest score (using the upper and lower real limits of the range).

Range

- URL x_{max} - LRL x_{min}
  - e.g. 3, 7, 12, 8, 5, 10

Problems?

- Distribution 1
  - 1, 8, 9, 9, 10, 10 R = ?
- Distribution 2
  - 1, 2, 3, 6, 8, 10 R = ?

The Interquartile Range

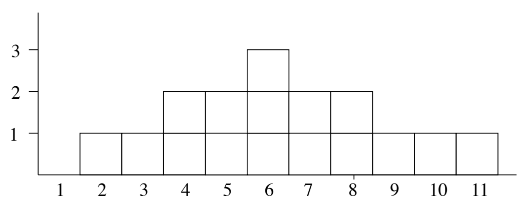
- The **interquartile range** is the distance covered by the middle 50% of the distribution (the difference between Q1 and Q3).

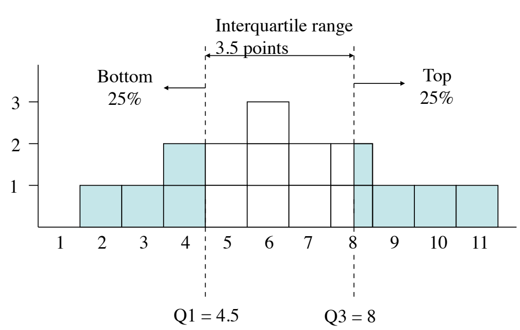
Scores

2, 3, 4, 4, 5, 5, 6, 6,

6, 7, 7, 8, 8, 9, 10, 11

x	f	cf	cp	c%
11	1	16	16/16	100%
10	1	15	15/16	93.75%
9	1	14	14/16	87.5%
8	2	13	13/16	81.25%
7	2	11	11/16	68.75%
6	3	9	9/16	56.25%
5	2	6	6/16	37.5%
4	2	4	4/16	25%
3	1	2	2/16	12.5%
2	1	1	1/16	6.25%



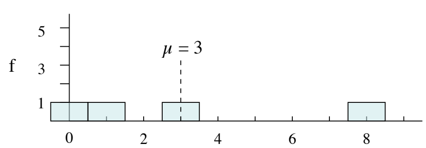


The Standard Deviation

- **Standard deviation** measures the **standard** (or average) distance between a score and the mean.

0, 1, 3, 8 $\quad \mu = \frac{8+1+3+0}{4} = 3$

x	(x- μ)
8	8 - 3 = +5
1	1 - 3 = -2
3	3 - 3 = 0
0	0 - 3 = -3



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x	x - μ	(x - μ)^2
1	1 - 2 = -1	1
0	0 - 2 = -2	4
6	6 - 2 = +4	16
1	1 - 2 = -1	1

$\sum x = 8$

$\mu = 2$

$22 = \sum(x - \mu)^2 = SS$

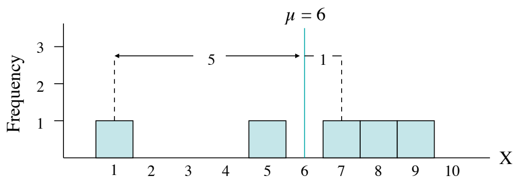
or

x	x^2
1	1
0	0
6	36
1	1

$\sum x = 8$

$\sum x^2 = 38$

$$SS = \sum x^2 - \frac{(\sum x)^2}{N}$$
$$= 38 - \frac{8^2}{4}$$
$$= 38 - 16$$
$$= 22$$



- 1, 9, 5, 8, 7
- $\mu = 6$

x	(x - μ)	(x - μ)^2
1	1 - 6 = -5	25
9	9 - 6 = +3	9
5	5 - 6 = -1	1
8	8 - 6 = +2	4
7	7 - 6 = +1	1

$$\sum(x - \mu)^2 = 40 = SS$$

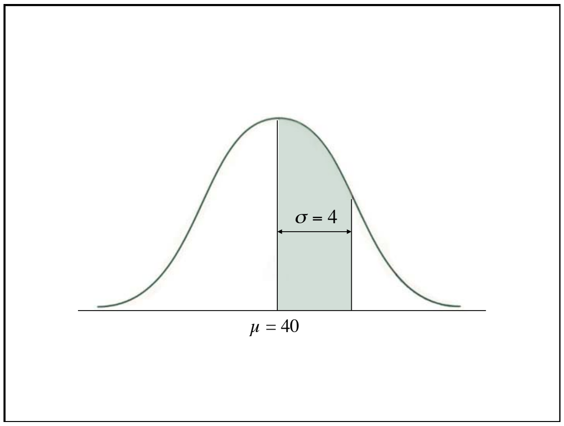
$$\sigma^2 = \frac{SS}{N} = \frac{\sum(x - \mu)^2}{N} = \frac{40}{5} = 8$$

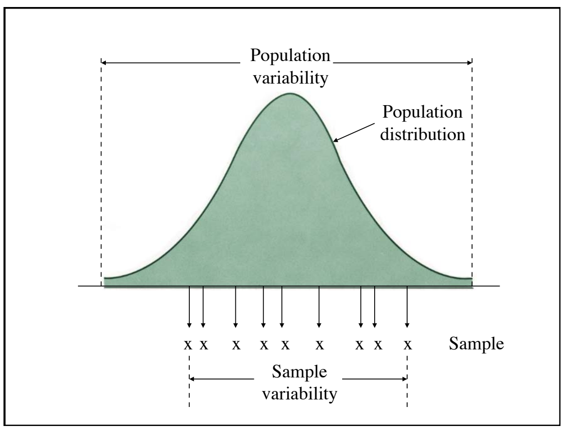
$$\sigma = \sqrt{\frac{SS}{N}} = \sqrt{\frac{\sum(x - \mu)^2}{N}} = 2.83$$

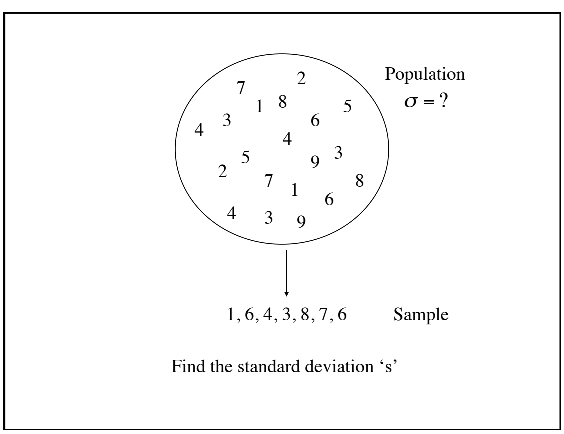
Variance and Standard Deviation for a population of scores

$$\sigma^2 = \frac{SS}{N} = \frac{\sum(x - \mu)^2}{N}$$

$$\sigma = \sqrt{\frac{SS}{N}} = \sqrt{\frac{\sum(x - \mu)^2}{N}}$$


$\sigma = 4$
$\mu = 40$


Population variability
Population distribution
x x x x x x x x x
Sample
Sample variability


7 2 1 8 5 4 3 6 4 5 9 3 2 7 1 8 6 4 3 9
Population
$\sigma = ?$
1, 6, 4, 3, 8, 7, 6
Sample
Find the standard deviation 's'

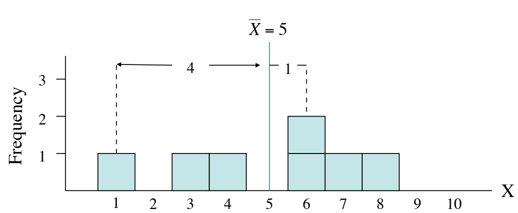
Variance and Standard Deviation for a Sample Used to Estimate the Population Value

Variance:

$$s^2 = \frac{SS}{n-1} = \frac{\sum(x-\bar{x})^2}{n-1}$$

$$s = \sqrt{\frac{SS}{n-1}} = \sqrt{\frac{SS}{n-1}}$$

1, 6, 4, 3, 8, 7, 6,



1, 6, 4, 3, 8, 7, 6

Sample

$\overline{X} = \frac{\sum x}{n} = \frac{35}{7} = 5$

x	$(x-\overline{X})$	$(x-\overline{X})^2$
1	1 - 5 = -4	16
6	6 - 5 = +1	1
4	4 - 5 = -1	1
3	3 - 5 = - 2	4
8	8 - 5 = +3	9
7	7 - 5 = +2	4
6	6 - 5 = +1	1

$\sum(x-\overline{X})^2 = SS = 36$

variance $s^2 = \frac{\sum(x-\overline{X})^2}{n-1}$ or $\frac{SS}{n-1}$

standard deviation $s = \sqrt{\frac{\sum(x-\overline{X})^2}{n-1}} = \sqrt{\frac{36}{6}} = \sqrt{6} = 2.45$ or $\sqrt{\frac{SS}{n-1}}$

Sum of Squares

$$SS = \sum(x-\overline{X})^2$$

$$s^2 = \frac{\sum(x-\overline{X})^2}{n-1}$$

$$s = \sqrt{\frac{\sum(x-\overline{X})^2}{n-1}}$$

But Also:

$$SS = \sum x^2 - \frac{(\sum x)^2}{n}$$

x	x^2
1	1
6	36
4	16
3	9
8	64
7	49
6	36
35	211

$$SS = \sum x^2 - \frac{(\sum x)^2}{n}$$

$$= 211 - \frac{35^2}{7}$$

$$= 211 - \frac{1225}{7}$$

$$= 211 - 175 \quad = 36$$

$$\sigma^2 = \frac{SS}{N} = \frac{\sum(x-\mu)^2}{N}$$

$$\sigma = \sqrt{\frac{SS}{N}} = \sqrt{\frac{\sum(x-\mu)^2}{N}}$$

$$s^2 = \frac{SS}{n-1} = \frac{\sum(x-\overline{X})^2}{n-1}$$

$$s = \sqrt{\frac{SS}{n-1}} = \sqrt{\frac{\sum(x-\overline{X})^2}{n-1}}$$

Example

- Randomly select a score from a population

  x = 47

- What value would you predict for the population mean?

  if $\sigma = 4$

  if $\sigma = 20$

Properties of the Standard Deviation

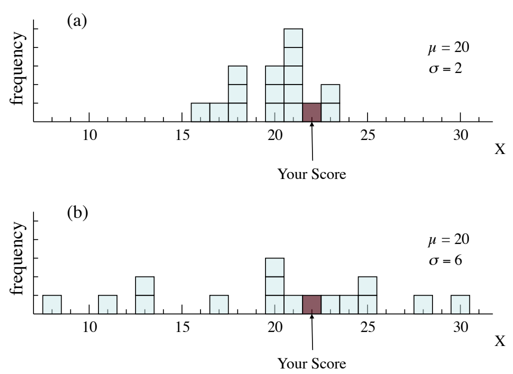
1. The same score can have very different meanings in 2 different distributions
2. Standard deviation helps us make predictions about sample data

   e.g. Figure 4.8 — low variability / high variability

   What is the probability of picking a score near μ = 20 ?

3. Sampling error - how big?

   (standard deviation a measure)



Transformations of Scale

1. Adding a constant to each score will not change the standard deviation
2. Multiplying each score by a constant causes the standard deviation to be multiplied by the same constant

Comparing Measures of Variability

- Two considerations determine the value of any statistical measurement:

1. The measures should provide a stable and reliable description of the scores. It should not be greatly affected by minor details in the set of data.
2. The measure should have a consistent and predictable relationship with other statistical measurements.

Factors that Affect Variability

1. Extreme scores
2. Sample size
3. Stability under sampling
4. Open-ended distributions

Relationship with Other Statistical Measures

- Variance and standard deviation are mathematically related to the mean. They are computed from the squared deviation scores (squared distance of each score from the mean).
- Median and semi-interquartile range are both based on percentiles and therefore are used together. When the median is used to report central tendency, semi-interquartile range is often used to report variability.
- Range has no direct relationship to any other statistical measure.

Sample variability and degrees of freedom

df = n - 1

The Mean and Standard Deviation as Descriptive Statistics

- If you are given numerical values for the mean and the standard deviation, you should be able to construct a visual image (or a sketch) of the distribution of scores.
- As a general rule, about 70% of the scores will be within one standard deviation of the mean, and about 95% of the scores will be within a distance of two standard deviations of the mean.

Mean number of errors on easy vs. difficult tasks for males vs. females

	Easy	Difficult
Female	1.45	8.36
Male	3.83	14.77

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When we report descriptive statistics for a sample, we should report a measure of central tendency and a measure of variability.

Mean number of errors on easy vs. difficult tasks for males vs. females

	Easy	Difficult
Female	M =1.45 SD = .92	M = 8.36 SD = 2.16
Male	M =3.83 SD =1.24	M =14.77 SD = 3.45

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