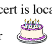
What is Problem Solving?

- We may consider a person to have a problem when: he or she wishes to attain goal for which no simple, direct means known. Examples:
  - Solve the crossword puzzle in today's newspaper 
  - Get my car running again 
  - Solve the statistics problems assigned by my Stats teacher $\sum x^2 - \frac{(\sum x)^2}{N} = ?$
  - Feed the hungry 
  - Find out where the arena for the concert is located
  - Get a birthday present for my mother 

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4 Aspects to a Problem:

- **Goal** - state of knowledge toward which the problem solving is directed
  - house designed properly
  - math equation completed
- **Givens** - objects, conditions, and constraints that are provided with the problem -- either explicitly or implicitly
  - Math word problem - supplies objects and initial conditions
  - Architectural design problem -- perhaps only some conditions (space, cost) provided
- **Means of Transformation**- ways to change the initial states
  - apply mathematical knowledge, architectural principles
- **Obstacles** - steps unknown, goal can't be directly achieved
  - Retrieval from memory not problem, but determining what procedure to apply, what principle can be used, etc - each obstacles

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Types of Problems

- **Well-defined Problems**
  - All 4 aspects of the problem specified
    - Tower of Hanoi
    - Mazes
    - 573 subtract 459
    - Drive to Chicago with complete directions
- **Ill-defined Problems**
  - One or more aspects of the problem not completely specified
    - Eradicate a dangerous disease
    - Capture and Punish Osama bin Laden
    - Bring an end to international terrorism
    - Having an interesting career

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Methods for Studying Problem Solving

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- **Intermediate Products**
  - Instead of recording only final answer to problem
  - Observe intermediate states on way to goal
  - Puzzles: Various moves
  - Math problems: Collect/analyze equations and other information written down
  - Constraints on explanations
- **Verbal Protocols**
  - Ask subjects to "think aloud" while performing task (solving problem)
  - Think-aloud versus Retrospective Reports
  - Reveal products of thought not the processes
- **Computer Simulation**
  - Build computer simulation based on protocols
  - Protocols supply products; Computer program supplies hypothesized processes
  - Must specify initial state, givens, transformations, and goal to computer to get it to perform as people do
  - Information processing limitations
  - Compare performance of program and person

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Problem Solving as Representation and Search:

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- **Tower of Hanoi Problem**- 3 pegs and 3 disks of different sizes
  - Initial State: 3 disks on peg 1, smallest on top, mid-size on middle peg, and largest on the bottom
  - Goal State: 3 disks on peg3, in same order as before (smallest on top)
  - Transformation Rules: Only 1 disk moved at a time and cannot put a larger disk on a smaller disk
- **What do you Need to do to solve this problem?**
  - 1) Keep track of current situation (which disks are on which pegs)
  - 2) For each configuration you need to consider possible moves to reach solution (goal state)
- **Challenge for Any Theory of Problem Solving**
  - How are the problem and the various possible configurations represented? (i.e. how does a person take the (incomplete) info in problem, elaborate and represent it?)
  - How is this representation operated on to allow problem solver to consider possible moves?

7

7

Newell and Simon (1972)

- **Information Processing System (i.e.** processing & storage limitations of **Problem Solver)**
  1. Information processed serially
  2. Limited capacity STM
  3. Unlimited LTM but takes time to access
- **Task Environment**
  1. Objective problem presented (not the internal representation)
  2. Task environment influences the internal representation
- **Problem Space**
  - Problem solver's internal representation of the problem
  - Problem States--Knowledge available to the problem solver at a given time (e.g. current situation, past situations, and/or guesses about future situations)
  - Problem Operators--Means of moving from one state to another
  - Problem Space Graph--A map of the problem space where locations are the states & the paths are the operators

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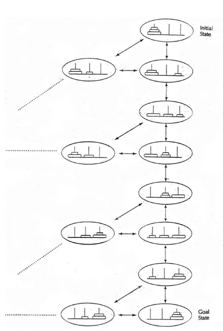

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Figure 12.8 The four-disk version of the Tower of Hanoi problem.

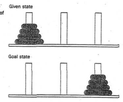
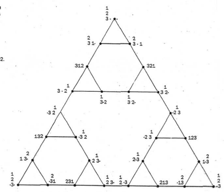


Figure 12.9 Problem graph of the legal moves and states for the three-disk Tower of Hanoi problem. Disks are represented by the numbers 1, 2, and 3, 1 being the smallest disk. *Source:* From Karat, 1982.



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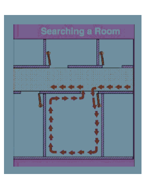
Problem Solving as Search

- **Search for a path through the problem space that connects the <u>initial state</u> to the <u>goal state</u>**
- **Objective problem space can be large**
- **<u>How to Search?</u>**
  - **Algorithm -** Systematic procedure guaranteed to lead to a solution
    - Exhaustive Search--e.g. explore all possible moves in Tower of Hanoi
    - Maze algorithm
    - Sometimes useful but also combinatorial explosions occur (e.g. chess)
  - **Heuristics -** Strategies used to guide search so that a complete search is not needed
    - No guarantee of solution but good chance of success with less effort
    - Best first search
    - Hill Climbing
    - Means Ends Analysis
    - Working Backwards

11

11

Fire Fighter Search Instruction



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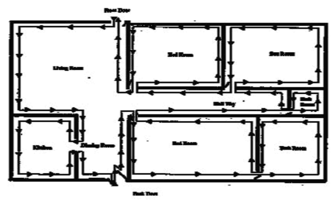
Fire Fighters: How to Search



In the diagram above it shows what is called a left hand search. This is used as a search technique when during rescue operations. The decision of either left or right hand search is made by the lead firefighter entering the house. There is no rhyme or reason to picking either left or right. However, when the decision is made usually a firefighter will search in the direction of the fire. In this diagram if the fire was in the kitchen we would do a left hand search as seen in the above diagram. If the fire was in last bedroom on the left then we would start a right hand search which would lead the firefighter to the bedroom quicker. During the search if right or left hand search is called, the firefighter never let go of the wall that is called. This is to orient the firefighter in a dark area.

13

13

Heuristic Search

- **Hill Climbing**
  - Plan one step ahead
  - Distance to goal guides search
  - Local versus global maximum
  - Sometime may not achieve solution (SF example)
- **Means-Ends Analysis**
  - Planning Heuristic (look ahead)
  - Steps
    1. Set up goal or subgoal
    2. Look for largest difference between current state & goal/subgoal state
    3. Select best operator to remove/reduce difference (e.g. set new subgoal)
    4. Apply operator
    5. Apply steps 2 to 4 until all subgoals & final goal achieved
  - Tower of Hanoi Example
  - San Francisco Example

14

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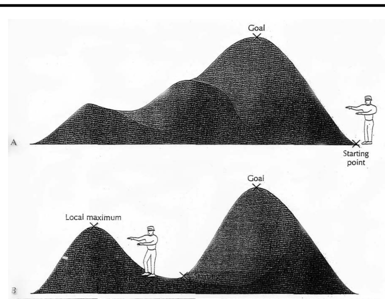
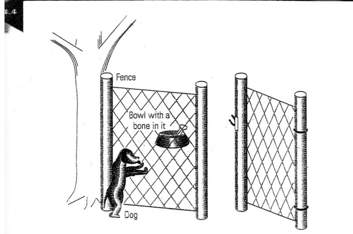


Figure 12.3 **Illustration of Hill-Climbing Heuristic** The terrain represents the closeness to problem solution, with higher elevation being closer to the goal. Panel A shows a blindfolded person moving steadily up the hill by feeling the terrain. Panel B shows the problem of local maximum for this heuristic.

15

15

Limits on the Hill-Climbing Strategy



According to the hill-climbing strategy, the dog should, at each step, choose a path that moves it closer and closer to the goal. However, this strategy will *fail* here, since the dog needs first to move *away* from the bone in order to reach the bone.

16

16

Planning Heuristic - means-ends analysis

- **Goal to get to San Francisco from NY**
  - 1.1) biggest distance - 3000 miles - best operator - airplane. Set goal- airport
    - 2.1) Current biggest distance - from current location to airport - best operator taxi. Set goal to get to taxi
      - 3.1) Current biggest distance - to taxi - best operator -walk. Set goal -walk
      - 3.2) Goal of walk to taxi area achieved
    - 2.2) State - at taxi - Goal of take taxi achieved
  - 1.2) State at airport - Goal to get to airport achieved
- **Goal to get to San Francisco achieved**

17

17

Disadvantages of Means-Ends Analysis

- Failure to find an operator to reduce a difference
- Sometimes must return to Initial State of Problem

18

18

The Water Lilies Problem

Water lilies are growing on Blue Lake. The water lilies grow rapidly, so that the amount of water surface covered by lilies *doubles* every 24 hours.

On the first day of summer, there was just one water lily. On the 90th day of the summer, the lake was entirely covered. On what day was the lake *half covered*?

- *Hint:*
- *Working backward from the goal is useful in solving this problem.*

19

19

Problem Solving as Representation

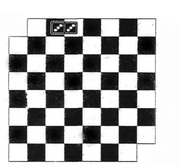
- Representation of the Problem is the Problem Space
- Why Representation Matters
  - Incomplete information (if certain information missing problem may be impossible to solve)
  - Combinatorial Complexity (some representations may make it difficult to apply operators & evaluate moves)
  - Some representations allow problem solver to apply operators easily and traverse the problem space in an efficient way; other representations do not
- Mutilated Checkerboard Problem
- Other Examples of Representation Effects
- Changing Representations to Solve Problems

20

20

The Mutilated Checkerboard Problem

- A checkerboard contains 8 rows and 8 columns, or 64 squares in all. You are given 32 dominoes, and asked to place the dominoes on the checkerboard so that each domino covers two squares. With 64 squares and 32 dominoes, there are actually many arrangements of dominoes that will cover the board.
- We now take out a knife, and cut away the top-left and bottom-right squares on the checkerboard. We also remove one of the dominoes. Therefore, you now have 31 dominoes which to cover the remaining 62 squares on the checkerboard. Is there an arrangement of the 31 dominoes that will cover the 62 squares? Each domino, as before, must cover two adjacent squares on the checkerboard.



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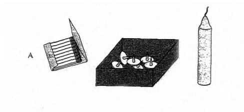
Duncker's Candle Problem



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22

Representations of Problems

Source: Based on Schwartz, 1971.

Read the following information, fill in the information in the matrix, and then answer the question, "What disease does Ms. Anderson have, and in what room is she?" (The answer is at the end of the chapter.)

Five people are in a hospital. Each person has only one disease, and each has a different disease. Each one occupies a separate room; the room numbers are 101 through 105.

1. The person with asthma is in Room 101.
2. Ms. Lopez has heart disease.
3. Ms. Green is in Room 105.
4. Ms. Smith has tuberculosis.
5. The woman with mononucleosis is in Room 104.
6. Ms. Thomas is in Room 101.
7. Ms. Smith is in Room 102.
8. One of the patients, other than Ms. Anderson, has gall bladder disease.

	Room Number				
	101	102	103	104	105
Anderson					
Lopez					
Green					
Smith					
Thomas					

23

23

The Buddhist Monk Problem

Exactly at sunrise one morning, a Buddhist monk set out to climb a tall mountain. The narrow path was not more than a foot or two wide, and it wound around the mountain to a beautiful, glittering temple at the mountain peak.

The monk climbed the path at varying rates of speed. He stopped many times along the way to rest and to eat the fruit he carried with him. He reached the temple just before sunset. At the temple, he fasted and meditated for several days. Then he began his journey back along the same path, starting at sunrise and walking, as before, at variable speeds with many stops along the way. However, his average speed going down the hill was greater than his average climbing speed.

Prove that there must be a spot along the path that the monk will pass on both trips at exactly the same time of day. (The answer is found in Figure 10.1.)

24

24

The Bookworm Problem

Solomon is proud of his 26-volume encyclopedia, placed neatly, with the volumes in alphabetical order, on his bookshelf. Solomon doesn't realize that there is a bookworm sitting on the front cover of the "A" volume. The bookworm begins chewing his way through the pages, on the shortest possible path toward the back cover of the "Z" volume.

Each volume is 3 inches thick (including pages and covers), so that the entire set of volumes requires 78 inches of bookshelf. The bookworm chews through the pages + covers at a steady rate of 3/4 of an inch per month. How long will it take before the bookworm reaches the back cover of the "Z" volume?

Hint: people who try an algebraic solution to this problem often end up with the wrong answer.

25

25

Improving Problem Solving by Focusing on Representation

- Examples:
  1. Use Images or Pictures (e.g. Bookworm problem and the Buddhist monk)
  2. Draw Diagrams (e.g. physics problems or missionaries & cannibals)
  3. Use Symbols to represent unknown quantities (e.g. math problems)
  4. Use Hierarchies (to represent relationships--e.g. a family tree)
  5. Use Matrices (to represent multiple constraints--e.g. the hospital problem or your class schedule)

26

26

Problem Solving Using Analogy (1)

- General importance of Analogy
  - Important component of intelligence
  - Teaching tool (e.g. atom as a miniature solar system)
- Using previous problem to solve new problem
- Dunker's Tumor Problem
  - Low convergence solution rate -- 10%
  - Following similar Fortress Problem (Gick & Holyoak, 1980, 1983)
    - 30% solution rate
    - 80% solution (with hint to use Fortress Problem)
- Failure to access relevant knowledge but success with hint. Why?

27

27

The Tumor Problem (Dunker, 1945; Gick & Holyoak (1980, 1983)

- Suppose you are a doctor faced with a patient who has a malignant tumor in his stomach. It is impossible to operate on the patient, but unless the tumor is destroyed the patient will die. There is a kind of ray that can be used to destroy the tumor. If the rays reach the tumor all at once at a sufficiently high intensity, the tumor will be destroyed. Unfortunately, at this intensity the healthy tissue that the rays pass through on the way to the tumor will also be destroyed. At lower intensities the rays are harmless to healthy tissue, but they will not affect the tumor either.
- What type of procedure might be used to destroy the tumor with the rays, and at the same time avoid destroying the healthy tissue?
- **One solution**:

28

28

The General and Fortress Problem (after Gick & Holyoak 1980, 1983)

A small country was ruled from a strong fortress by a dictator. The fortress was situated in the middle of the country, surrounded by farms and villages. Many roads led to the fortress through the countryside. a rebel general vowed to capture the fortress. The general knew that an attack by his entire army would capture the fortress. He gathered his army at the head of one of the roads. The mines were set so that small bodies of men could pass over them safely, since the dictator needed to move his troops and workers to and from the fortress. However, any large force would detonate the mines. Not only would this blow up the road, but it would also destroy many neighboring villages. It therefore seemed impossible to capture the fortress.

What is the solution?

29

29

Problem Solving Using Analogy (2)

- **Terminology**
  - Problem isomorphs
  - Target versus Source Problem
  - Surface versus Structural Features
- **Failures to solve problem isomorphs**
- **Attention to surface features/content rather than abstract, underlying structure**
- **Content-dependent storage**--(e.g. presented with 'tumor' problem people look for info about tumors)
- **Strategies to improve use of Analogy:**
  - Goal: access relevant abstract knowledge
  - Provide training on multiple convergence type problems before target
  - Encourage comparison of multiple source problems
  - Increase understanding of source problem (e.g. understanding of goal structure & why each step taken)
  - Other research on self-explanations (e.g. Chi, et al, 1994)

30

30

Table 12.3 *Correspondences among the military problem, the radiation problem, and the convergence schema*

Military problem
- Initial state
  - Goal: Use army to capture fortress.
  - Resources: Sufficiently large army.
  - Constraint: Unable to send entire army along one road.
- Solution plan: Send small groups along multiple roads simultaneously.
- Outcome: Fortress captured by army.

Radiation problem
- Initial state
  - Goal: Use rays to destroy tumor.
  - Resources: Sufficiently powerful rays.
  - Constraint: Unable to administer high-intensity rays from one direction.
- Solution plan: Administer low-intensity rays from multiple directions simultaneously.
- Outcome: Tumor destroyed by rays.

Convergence schema
- Initial state
  - Goal: Use force to overcome a central target.
  - Resources: Sufficiently great force.
  - Constraints: Unable to apply full force along one path.
- Solution plan: Apply weak forces along multiple paths simultaneously.
- Outcome: Central target overcome by force.

SOURCE: From "Schema induction and analogical transfer," by M. L. Gick and K. Holyoak, 1983, *Cognitive Psychology*, 15, 1–38.

31

31

Research suggests people more likely to use analogies effectively under following circumstances:

1. When instructed to compare 2 problems that initially seem unrelated because they have different surface structures
2. When shown several structurally similar problems before tackling target problem
3. When they try to solve the source problem, rather than simply looking at source problem
4. When given hint that strategy used on a specific earlier problem may also be useful in solving target problem

32

32

Additional Factors that Influence Problem Solving

- Mental Set
- Functional Fixedness
- Insight versus Non-insight Problems

33

33

Mental Set and Functional Fixedness

- **Mental Set**
  - Attempt to apply previous problem method to new problems that could be solved with easier method
  - Classic example: Luchin's Water Jar Problem (1942)
  - First 5 problems solved using B with A & C
  - People persist in solving problems 7-8 same way missing much easier solution
  - Links to creativity
- **Functional Fixedness**
  - Rely too heavily on previous knowledge about conventional uses of objects
  - Classic example: Duncker's Candle Problem
  - People don't think to use the box (which contains the tacks) for another purpose
  - Box not included in the representation (problem space)
  - Must think flexibly about new ways to use objects
  - Personal example: My W-2 for my tax return in Morocco

34

34

Luchin's Water Jar Problem

Imagine that you have three jars, A, B, and C. For each of the seven problems below, the capacity of the three jars is listed. You must use these jars in order to obtain the amount of liquid specified in the Goal column. You may obtain the goal amount by adding or subtracting the quantities listed in A, B, and C. (The answers can be found a little later in the text, in the discussion of mental set.)

Problem	A	B	C	Goal
1	24	130	3	100
2	9	44	7	21
3	21	58	4	29
4	12	160	25	98
5	19	75	5	46
6	23	49	3	20
7	18	48	4	22

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Luchin's Water Jar Problem

Problem	A	B	C	Goal
1	24	130	3	100
2	9	44	7	21
3	21	58	4	29
4	12	160	25	98
5	19	75	5	46
6	23	49	3	20
7	18	48	4	22

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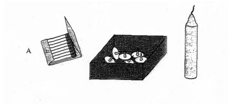
Duncker's Candle Problem



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37

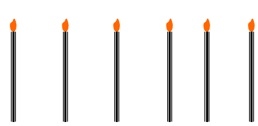
Insight versus Non-Insight Problem Solving

- **Insight problem** initially seems impossible to solve (no progress) and then suddenly solved, often by perceiving new relations amongst the objects in the problem
- **Non-Insight problems** solved in gradual fashion (e.g. Tower of Hanoi)
- **Classic Insight Problem**: Kohler's research with chimpanzees during WWI on island of Teneriffe:
- **Sudden perception of solution** often achieved by **change** in the **representation** of problem
- **Inappropriate assumptions**
  - **Examples**:
    - Six matches to form 4 equilateral triangles
    - Nine dot Problem
- **Metacognition** during Problem Solving
- **Role of Language** in Problem Solving

38

38

6 Matches Problem



Can you make 4 equilateral triangles?

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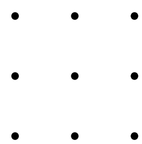
Nine Dot Problem

Draw no more than 4 straight lines (without lifting the pencil from the paper) that cross through all nine dots



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Coin Problem

A stranger approached a museum curator and offered him an ancient bronze coin. The coin had an authentic appearance and was marked with the date 544 B.C. The curator had happily made acquisitions from suspicious sources before, but this time he promptly called the police and had the stranger arrested. Why?

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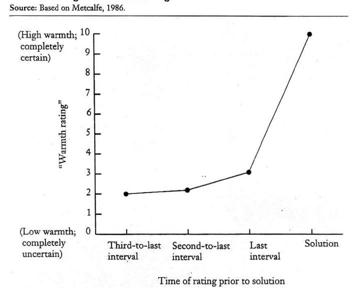
"Warmth Ratings" for Answers That Were Correct, as a Function of Time of Rating Prior to Answering.
Source: Based on Metcalfe, 1986.



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Incubation

- Definition & Background
  - Process by which if you reach an impasse in solving a problem, taking a break (during which you don't work on the problem) & then trying later, you're more likely to solve problem
  - Controversial claim
  - Informal versus Controlled Research
- Why Incubation might help
  - Break mental set or functional fixedness
  - May encourage change of problem representation
- Issues
  - How to know what the problem solver does during break
  - Interesting issue
  - Compare with distributed practice
  - Relevance to insight problem solving

43

43

Suggestions for Improving Problem Solving

(from Ashcraft's Fundamental's of Cognition p. 412)

1. Increase your domain knowledge
2. Automate some components of the problem-solving solution
3. Follow a systematic plan
4. Draw inferences
5. Develop sub-goals
6. Work backward
7. Search for contradictions
8. Search for relations among problems
9. Find a different problem representation
10. If all fails, try practice.

44

44

Man at Home Problem

- There is a man at home. The man is wearing a mask. There is a man coming home.
- What is happening here?

45

45